The LARES mission revisited: an alternative scenario

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Abstract

In the original LARES mission the general relativistic Lense–Thirring effect would be detected by using as observable the sum of the residuals of the nodes of the existing passive geodetic laser-ranged LAGEOS satellite and of its proposed twin LARES. The proposed nominal orbital configuration of the latter one would reduce the systematic error due to the mismodelling in the even zonal harmonics of the geopotential, which is the main source of error, to 0.3%, according to the most recent Earth gravity model EGM96. This observable turns out to be sensitive to possible departures of the LARES orbital parameters from their nominal values due to the orbital injection errors. By adopting a suitable combination of the orbital residuals of the nodes of LAGEOS, LAGEOS II and LARES and the perigees of LAGEOS II and LARES it should be possible to reduce the error due to the geopotential by one order of magnitude, according to the EGM96 gravity model. Moreover, the sensitivity to the orbital injection errors should be greatly reduced. According to a preliminary estimate of the error budget, the total error of the experiment should be reduced to less than 1%. In the near future, when the new data of the terrestrial gravitational field from the CHAMP and GRACE missions will be available, a further increase in the accuracy should be obtained. The proposal of placing LARES in a polar 2,000 km altitude orbit and considering only its nodal rate would present the drawback that even small departures from the polar geometry would yield notable errors due to the mismodelled even zonal harmonics of the geopotential, according to the EGM96 model.

1 Introduction

In its weak–field and slow–motion approximation General Relativity predicts that, among other things, the orbit of a test particle freely falling in the gravitational field of a central rotating body is affected by the so called gravitomagnetic dragging of the inertial frames or Lense–Thirring effect. More precisely, the longitude of the ascending node Ω and the argument of the perigee ω of the orbit [*Sterne*, 1960] undergo tiny precessions according to [*Lense and Thirring*, 1918]

$$\dot{\Omega}_{\mathrm{LT}} = \frac{2GJ}{c^2 a^3 (1-e^2)^{\frac{3}{2}}}, \tag{1}$$

$$\dot{\omega}_{\mathrm{LT}} = -\frac{6GJ\cos i}{c^2 a^3 (1-e^2)^{\frac{3}{2}}}, \tag{2}$$

in which G is the Newtonian gravitational constant, J is the proper angular momentum of the central body, c is the speed of light *in vacuum*, a, e and i are the semimajor axis, the eccentricity and the inclination, respectively, of the orbit of the test particle.

The first measurement of this effect in the gravitational field of the Earth has been obtained by analyzing a suitable combination of the laser-ranged data to the existing passive geodetic satellites LAGEOS and LAGEOS II [*Ciufolini et al.*, 1998]. The observable [*Ciufolini*, 1996] is a linear trend with a slope of 60.2 milliarcseconds per year (mas/y in the following) and includes the residuals of the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II[1]. The Lense–Thirring precessions for the LAGEOS satellites amount to

$$\dot{\Omega}_{\mathrm{LT}}^{\mathrm{LAGEOS}} = 31 \text{ mas/y}, \tag{3}$$

$$\dot{\Omega}_{\mathrm{LT}}^{\mathrm{LAGEOS\ II}} = 31.5 \text{ mas/y}, \tag{4}$$

$$\dot{\omega}_{\mathrm{LT}}^{\mathrm{LAGEOS}} = 31.6 \text{ mas/y}, \tag{5}$$

$$\dot{\omega}_{\mathrm{LT}}^{\mathrm{LAGEOS\ II}} = -57 \text{ mas/y}. \tag{6}$$

The total relative accuracy of the measurement of the solve-for parameter μ_{LT}, introduced in order to account for this general relativistic effect, is about $2-3 \times 10^{-1}$ [*Ciufolini et al.*, 1998].

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[1]The perigee of LAGEOS was not used because it introduces large observational errors due to the smallness of the LAGEOS eccentricity [*Ciufolini*, 1996] which amounts to 0.0045.

In this kind of experiments using Earth satellites the major source of systematic errors is represented by the aliasing trends due to the classical secular precessions [*Kaula*, 1966] of the node and the perigee induced by the mismodelled even zonal harmonics of the geopotential J_2, J_4, J_6,... Indeed, according to the present knowledge of the Earth's gravity field based on the EGM96 model [*Lemoine et al.*, 1998], the J_{2n} errors are comparable in size with the gravitomagnetic precessions of interest, especially for the first two even zonal harmonics. In the performed LAGEOS experiment the adopted observable allowed for the cancellation of the static and dynamical effects of J_2 and J_4. The remaining higher degree even zonal harmonics affected the measurement at a 13% level.

In order to achieve a few percent accuracy, in [*Ciufolini*, 1986] it was proposed to launch a passive geodetic laser-ranged satellite- the former LAGEOS III - with the same orbital parameters of LAGEOS apart from its inclination which should be supplementary to that of LAGEOS.

This orbital configuration would be able to cancel out exactly the classical nodal precessions, which are proportional to $\cos i$, provided that the observable to be adopted is the sum of the residuals of the nodal precessions of LAGEOS III and LAGEOS

$$\delta\dot{\Omega}^{\mathrm{III}} + \delta\dot{\Omega}^{\mathrm{I}} = 62\mu_{\mathrm{LT}}. \tag{7}$$

Later on the concept of the mission slightly changed. The area-to-mass ratio of LAGEOS III was reduced in order to make less relevant the impact of the non-gravitational perturbations, the total weight of the satellite was reduced to about 100 kg, i.e. to about 25% of the weight of LAGEOS, and the eccentricity was enhanced in order to be able to perform other general relativistic tests: the LARES was born [*Ciufolini*, 1998]. The orbital parameters of LAGEOS, LAGEOS II and LARES are in Tab. 1.

Table 1: Orbital parameters of LAGEOS, LAGEOS II and LARES.

Orbital parameter	LAGEOS	LAGEOS II	LARES
a (km)	12,270	12,163	12,270
e	0.0045	0.014	0.04
i (deg)	110	52.65	70

At present the LARES experiment is just at a Phase–A stage and has not yet been approved by any space agency. Although much cheaper than other proposed and approved complex space–based missions, funding is the major obstacle in implementing the LARES project.

In this paper we investigate the possibility of modifying the original LARES mission in order to achieve significant improvements in the reduction of some relevant systematic errors. The paper is organized as follows. In Section 2 we analyze in detail the impact of the unavoidable orbital injection errors in the orbital parameters of LARES on the systematic error induced by the mismodelling in even zonal harmonics of the geopotential according to the most recently released Earth gravity model EGM96. Moreover, in Section 3 we propose an alternative configuration which should be able to reduce this error by one order of magnitude. It adopts as observable a suitable combination of the orbital residuals of the nodes of LAGEOS, LAGEOS II and LARES, and the perigees of LAGEOS II and LARES. It presents also the important advantage that it is almost insensitive to the errors in the inclination of LARES, contrary to the original LAGEOS/LARES only configuration. A further observable, based only on the nodes of the three LAGEOS satellites and of Ajisai, is also presented. Some possible less convincing implications of placing the LARES in a low-altitude polar orbit are examined in Section 4. Section 5 is devoted to the conclusions.

2 The impact of the even zonal harmonics of the geopotential on the original LARES mission

Let us calculate the systematic error induced by the mismodelling in the even degree zonal coefficients J_2, J_4,... of the geopotential on the sum of the classical precessions of the nodes of LAGEOS and LARES. It is important to stress that it is the major source of systematic error and cannot be eliminated in any way. We will use the covariance matrix of the Earth gravity field model EGM96 [*Lemoine et al.*, 1998] by summing up in a root sum square fashion the correlated contributes up to degree $l = 20$. The relative error obtained by using the nominal values of Tab. 1 amounts to

$$\frac{\delta\mu_{\text{LT}}}{\mu_{\text{LT}}}_{\text{zonals}} = 3 \times 10^{-3}. \tag{8}$$

It is not equal to zero because we have assumed $e_{\mathrm{LR}} = 0.04$ while $e_{\mathrm{LAGEOS}} = 0.0045$. If it was $e_{\mathrm{LR}} = e_{\mathrm{LAGEOS}}$, then the classical nodal precessions would be exactly equal in value and opposite in sign and would cancel out. Note that the coefficients with which $\delta\dot{\Omega}^{\mathrm{III}}$ and $\delta\dot{\Omega}^{\mathrm{I}}$ enter the combination of eq. (7) do not depend on any orbital parameters: they are constant numbers equal to 1; this combination allows to cancel out all the classical nodal precessions due to the J_{2n} coefficients, including those induced by J_2 and J_4 which are cancelled out *a priori* in the combination used in the LAGEOS experiment [*Ciufolini*, 1996]. They are the most effective in aliasing the Lense–Thirring precessional rates.

Now we will focus on the sensitivity of $\frac{\delta\mu_{\mathrm{LT}}}{\mu_{\mathrm{LT}}}_{\mathrm{zonals}}$ to the possible orbital injection errors in the orbital parameters of LARES. For a former analysis see [*Casotto et al.*, 1990]. It is particularly interesting to consider the impact of the errors in the inclination and the semimajor axis. The ranges of variation for them have been chosen in a very conservative way in order to take into account low-precision and low-costs injection scenarios.
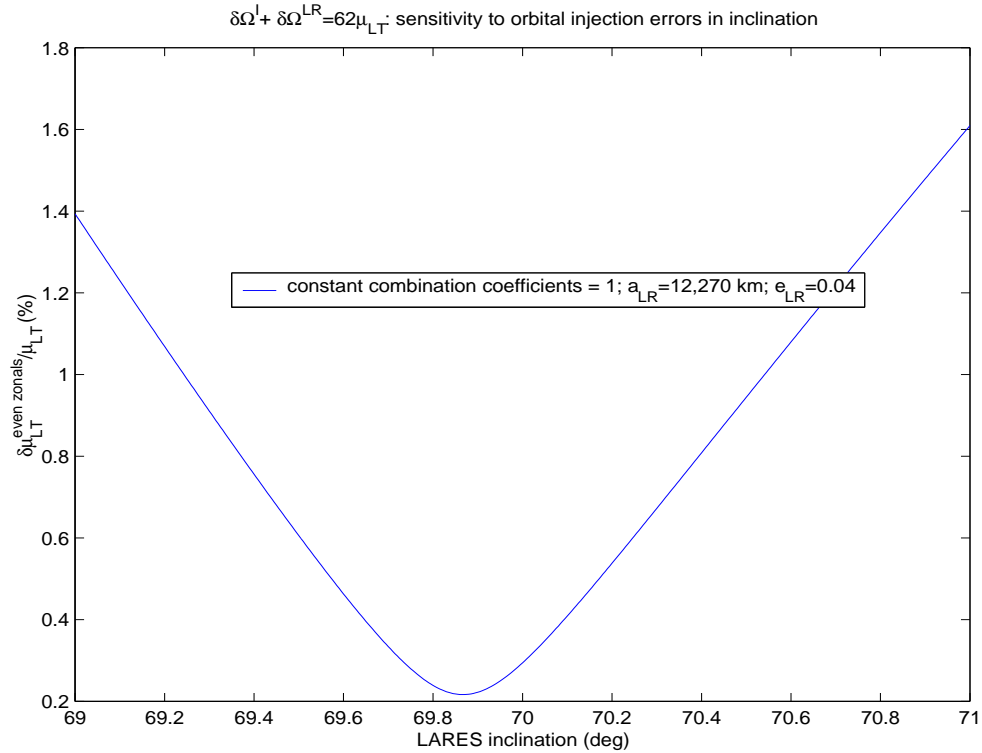

Figure 1: Influence of the injection errors in the LARES inclination on the error due to the even zonal harmonics of the geopotential.
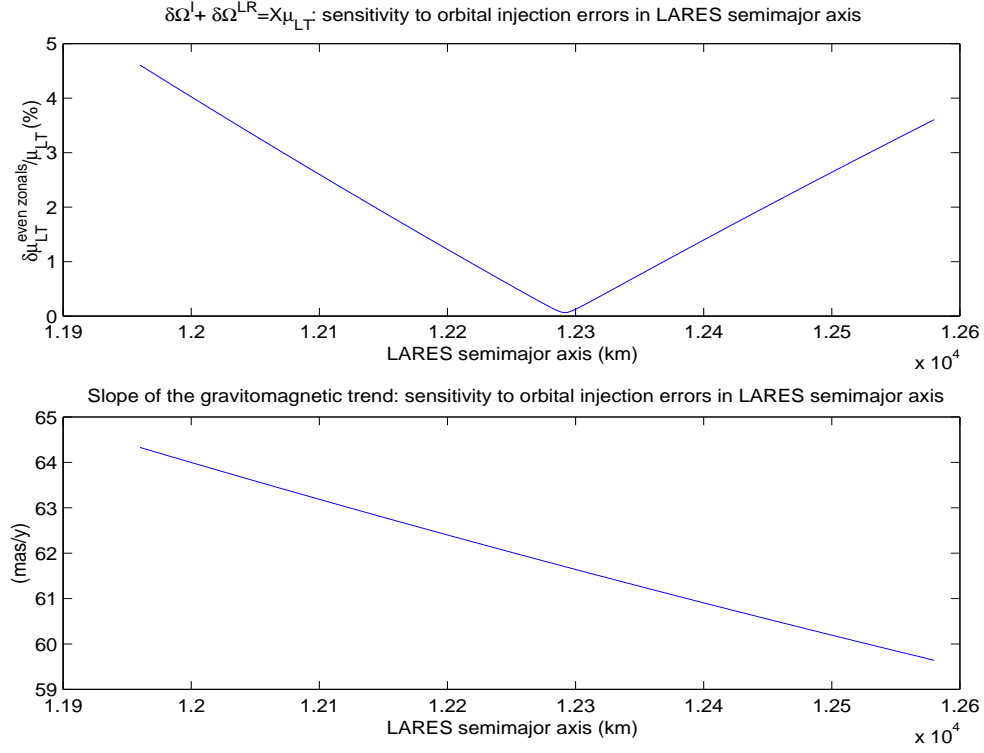
Figure 2: Influence of the injection errors in the LARES semimajor axis on the error due to the even zonal harmonics of the geopotential.

From Fig.1 it is interesting to note that the minimum value of the systematic zonal error, which amounts to 2.1×10^{-3}, does not correspond to $i_{\mathrm{LR}} = 70$ deg but it is obtained for a slightly smaller value. It is possible to show that for $e_{\mathrm{LR}} = e_{\mathrm{LAGEOS}}$ the minimum is 0 and that it is attained at $i_{\mathrm{LR}} = 70$ deg. The maximum error in Fig. 1 amounts to 1.6×10^{-2}. This suggests that the original LARES project is relatively sensitive to small departures of i_{LR} from its nominal value. Fig. 2 shows that even more relevant is the sensitivity to the LARES semimajor axis. Also in this case the minimum is attained at a value of a_{LR} smaller than the nominal $a_{\mathrm{LR}} = 12,270$ km. Notice that the variation of the error is more than one order of magnitude and may also reach values of some percent. For $e_{\mathrm{LR}} = e_{\mathrm{LAGEOS}}$ the minimum error amounts to 0 and it is obtained for $a_{\mathrm{LR}} = 12,270$ km, as expected. In obtaining Fig. 2 we have accounted for the dependence of the nodal Lense–Thirring precession on a by varying, accordingly, the slope of the general relativistic trend. The sensitivity to eccentricity variations is less relevant: e.g., by varying it from 0.03 to 0.05 the relative systematic zonal error increases

from 1.6×10^{-3} to just 4.6×10^{-3}.

3   An alternative LARES scenario

Here we will look for an alternative observable involving the orbital elements of LARES satisfying the following requirements

- It should yield a value for the systematic error due to the mismodelled even zonal harmonics of the geopotential smaller than that of the simple sum of the nodes of LAGEOS and LARES. Moreover, such error should be less sensitive to the departures of the possible real orbital elements of LARES from the nominal values of Tab. 1

- It should contain and, if possible, reduce, the time–varying gravitational and non–gravitational part of the error budget

These requirements could be implemented by setting up a suitable orbital combination which cancels out the contributions of as many mismodelled even zonal harmonics as possible, following the strategy of the LAGEOS experiment outlined in [*Ciufolini*, 1996]. To this aim we will consider only the satellites of the LAGEOS family, both because they are the best laser-ranged targets and because the gravitational and non-gravitational perturbations affecting their orbits have been, and will be, extensively and thoroughly analyzed. Moreover, since they are almost insensitive to the even zonal harmonics of the geopotential of degree higher than $l = 20$, the use of the covariance matrix of EGM96 up to[2] $l = 20$ should allow for realistic estimates of the systematic error due to the static part of the terrestrial gravitational field.

Our result is

$$\delta\dot{\Omega}^{\text{LAGEOS}} + c_1\delta\dot{\Omega}^{\text{LAGEOS II}} + c_2\delta\dot{\Omega}^{\text{LARES}} + c_3\delta\dot{\omega}^{\text{LAGEOS II}} + c_4\delta\dot{\omega}^{\text{LARES}} = 61.8\mu_{\text{LT}}, \qquad (9)$$

with

$$c_1 = 6 \times 10^{-3}, \qquad (10)$$

$$c_2 = 9.83 \times 10^{-1}, \qquad (11)$$

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[2]For higher degrees the reliability of the EGM96 model is questionable.

$$c_3 = -1 \times 10^{-3}, \tag{12}$$

$$c_4 = -2 \times 10^{-3}. \tag{13}$$

The coefficients c_i given by eqs. (10)-(13) have been obtained by solving for the five unknowns δJ_2, δJ_4 δJ_6, δJ_8 and μ_{LT} a nonhomogeneous algebraic linear system of five equations expressing the observed mismodelled classical precessions of the nodes of LAGEOS, LARES and LAGEOS II and the perigees of LAGEOS II and LARES. They depend on the orbital parameters of LAGEOS, LAGEOS II and LARES (nominal values of Tab. 1) and are built up so to cancel out all the static and dynamical contributions of degree $l = 2,\ 4,\ 6,\ 8$ and order $m = 0$ of the Earth' s gravitational field.

The relative systematic error due to the J_{2n}, $n \geq 5$, according to EGM96 up to degree $l = 20$, amounts to

$$\frac{\delta\mu_{\mathrm{LT}}}{\mu_{\mathrm{LT}}}_{\mathrm{zonals}} = 2 \times 10^{-4}, \tag{14}$$

which is one order of magnitude better than the result of eq. (8).

Fig. 3 and Fig. 4 show the achievements realized in reducing the sensitivity of the proposed combined residuals to the orbital injection errors in the LARES orbital elements. In obtaining Fig. 3 and Fig. 4 we have accounted for the dependence on a_{LR} and i_{LR} of both the coefficients and the Lense–Thirring precessions: it turns out that the variations in the slope of the general relativistic trend are very smooth with respect to the nominal value of 61.8 mas/y amounting to few mas/y and the values of the zonal error are much closer to the nominal one given by eq. (14). Also in this case, the minima are attained at slightly different values of the LARES orbital elements with respect to the nominal ones. It is also interesting to note in Fig. 3 that over a 3% variation of i_{LARES} the error due to the mismodelled zonal harmonics remain almost constant, while over a 5% variation of a_{LARES} it changes by 1 order of magnitude, as it turns out from Fig. 4. However, the result is quite satisfactory, especially if compared to Fig. 2.

3.1 Preliminary error budget estimate

It is worthwhile noticing that the time-varying gravitational and non–gravitational orbital perturbations which would affect the proposed combined residuals would be depressed by the small values of the coefficients with which some orbital elements enter the combination.
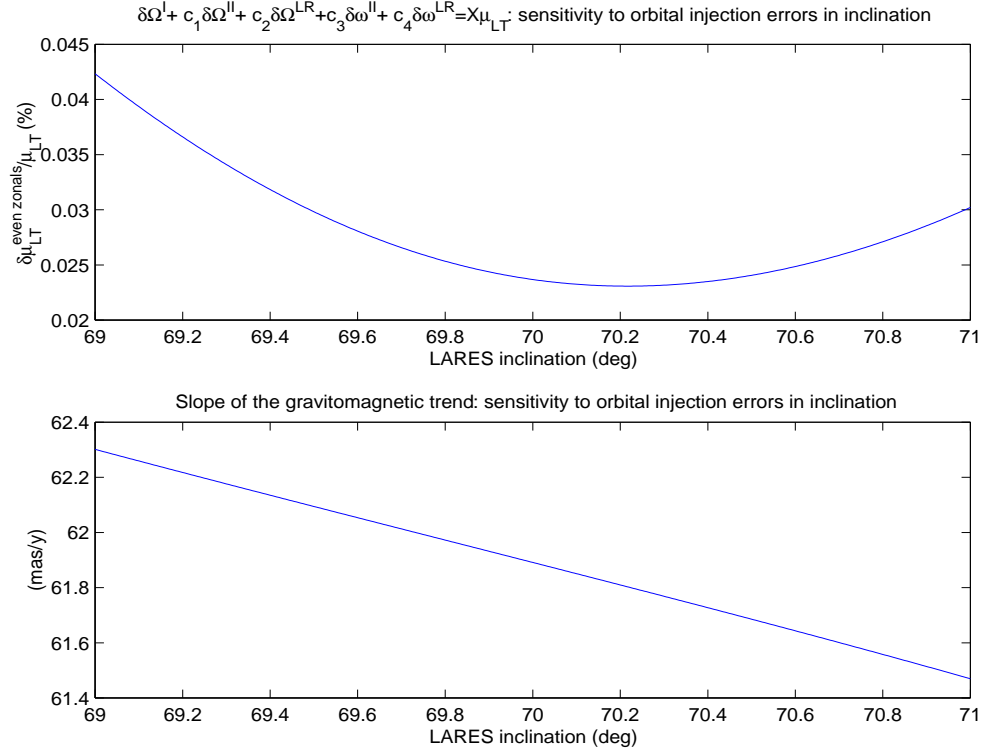
Figure 3: Alternative combined residuals: influence of the injection errors in the LARES inclination on the error induced by the even zonal coefficients of the geopotential.

- For example, in regard to the Earth solid and ocean tides [*Iorio*, 2001], it is important that the perigees of LAGEOS II and LARES, which are affected by very long–period uncancelled tidal perturbations, enter the combination weighted by small coefficients of order of 10^{-3}. On the contrary, the tidal perturbations which would affect the nodes of LAGEOS and LARES, which enter the combination with coefficients of the order of the unity, would have periods of short or medium length, so that they could be averaged out or, at least, could be viewed as empirically fitted quantities over an observational time span T_{obs} of few a years only [*Iorio and Pavlis*, 2001].

- More subtle and complex to model is the action of the non–gravitational (NG) perturbations. These perturbative effects depend on the physical and geometrical features of the satellites, on the geometry of their orbit in space – orientation and size – and on the complex interaction of the electromagnetic radiation of solar and terrestrial origin with the satellites' surfaces.

$\delta\Omega^I + c_1\delta\Omega^{II} + c_2\delta\Omega^{LR} + c_3\delta\omega^{II} + c_4\delta\omega^{LR} = X\mu_{LT}$: sensitivity to orbital injection errors in LARES semimajor axis

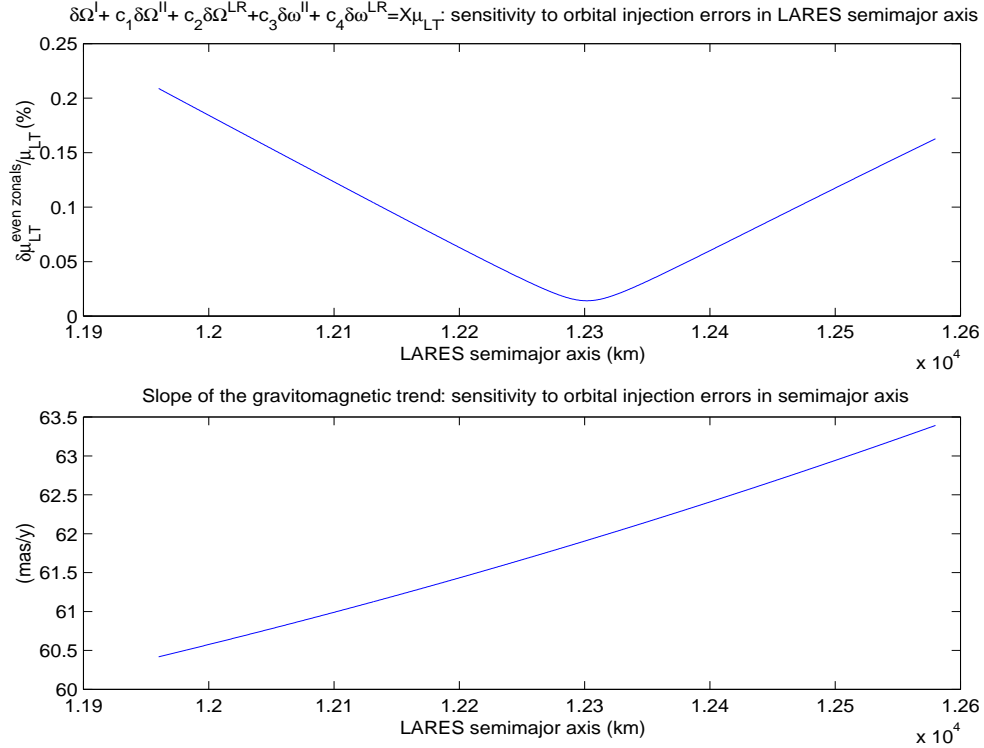Slope of the gravitomagnetic trend: sensitivity to orbital injection errors in semimajor axis

Figure 4: Alternative combined residuals: influence of the injection errors in the LARES semimajor axis on the error induced by the even zonal coefficients of the geopotential.

In particular thermal thrust effects – due to the uncertainties of some of their characteristic parameters – play a crucial role in the Lense-Thirring effect determination on both the node and the perigee of LAGEOS–type satellites [*Lucchesi*, 2001; 2002]. These perturbations, such as the solar Yarkovsky-Schach effect, the Earth Yarkovsky-Rubincam effect and the asymmetric reflectivity effect, are related to the satellite spin axis orientation and rate. This opens the problem of the satellites spin axis vector determination from ground observations in such a way to establish if the models developed for the spin axis evolution [*Bertotti and Iess*, 1991; *Habib et al.*, 1994; *Farinella et al.*, 1996; *Currie et al.*, 1997; *Bianco et al.*, 2001] are reliable or not. For example, the previously cited models seem no more able to explain the LAGEOS spin axis evolution starting from 1997 [*Métris et al.*, 1999]. Moreover, it should also be stressed that such subtle disturbing accelerations are included in a rather incomplete manner in the force models of orbit determination softwares like GEODYN II. It should be underlined that, if and when the

LARES mission will be implemented, the effects of the thermal forces will be different for the three satellites also because the dynamical states of their spins will radically differ from each other. Temporal variations in the reflectivity coefficients C_R of the LAGEOS satellites have also to be accurately determined and modelled using the orbital observations. Indeed, the reflectivity coefficient of the LAGEOS II satellite has been observed to be changing in time. However, this change can be accurately modelled from the experimental data, independently of the Lense–Thirring effect, because of the precise knowledge of the frequencies and relative amplitudes of the orbital perturbations due to the direct solar radiation pressure. In our error budget we have also included this source of error due to the mismodelling of time variations in the satellites reflectivity coefficients.

In the following we give the results – on the LT effect measurement – of a numerical simulation and analysis of the satellites orbit over a 7–year observational time span. We considered the previously quoted thermal thrust effects, the direct solar radiation pressure and Earth albedo on the satellites node and perigee rates. For the definition and characteristics of these perturbations we refer to literature, while for the most recent and significative results we refer to *Métris et al.*, [1997, 1999] and to *Lucchesi* [2001, 2002]. In Tab. 2, 3 and 4 the nominal results, from the numerical simulation and analysis, respectively in the case of LAGEOS, LAGEOS II and LARES are shown. The rates obtained are expressed in mas/y.

Table 2: Non–gravitational perturbations on LAGEOS node and perigee: nominal values.

Perturbation	$\dot{\Omega}$ (mas/y)	$\dot{\omega}$ (mas/y)
Yarkovsky-Rubincam	0.2	0.07
Yarkovsky-Schach	-0.04	-77
Asymmetric reflectivity	6×10^{-4}	52.9
Earth albedo	1.1	145
Direct solar radiation pressure	-7.3	-40,261

In the simulation performed we neglected the possibility of an asymmetric reflectivity effect in the case of LARES. This is due to the particular care with which its structure will be build–up (see LARES proposal, [*Ciufolini*, 1998]) to avoid some of the problems

Table 3:   Non–gravitational perturbations on LAGEOS II node and perigee: nominal values.

Perturbation	$\dot{\Omega}$ (mas/y)	$\dot{\omega}$ (mas/y)
Yarkovsky-Rubincam	-1.5	1
Yarkovsky-Schach	-0.5	150
Asymmetric reflectivity	3×10^{-3}	152
Earth albedo	-1.5	57
Direct solar radiation pressure	36	-2,693

Table 4:   Non–gravitational perturbations on LARES node and perigee: nominal values.

Perturbation	$\dot{\Omega}$ (mas/y)	$\dot{\omega}$ (mas/y)
Yarkovsky-Rubincam	-1.1	0.3
Yarkovsky-Schach	-9×10^{-4}	-2
Asymmetric reflectivity	–	–
Earth albedo	-1	24
Direct solar radiation pressure	8.5	-1,263

related with the thermal thrust effects, and also for the absence of the four Germanium cube–corner retroreflectors, which could be good candidates to explain at least a part of this anisotropy in the satellite hemispheres reflectivity. We also neglected the time variations of the reflectivity coefficients of the satellites. In the simulation we applied to LARES the spin model developed for LAGEOS in the *Farinella et al.* [1996] version (rapid–spin approximation)[3]. For LAGEOS we applied the model starting from 1993, when the gravitational torque gives the major contribution to the evolution. Of course, as previously stated, starting from 1997 the model does not give, for the spin components of LAGEOS, values in good agreement with the observations.

The larger effects in the analysed orbital elements are, of course, those due to the direct solar radiation pressure. This is a rather accurate modelled perturbative effect on passive satellites and we have assumed a value of about 0.5% for its uncertainty in the case of LAGEOS–type satellites, limited by the measurement errors in the determination of the

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[3]This model [*Lucchesi*, 2002] still gives very good results in the case of LAGEOS II, in agreement with the results of *Bianco et al.* [2001] for the satellite rotational rate.

solar constant $\Phi_\odot$ and especially in the knowledge of the satellite reflectivity coefficient C_R and in its temporal variation. Following a conservative approach we can assume an uncertainty of about 20% for the other perturbations. This is probably a pessimistic assumption, particularly for the perturbative effects of the Earth Yarkovsky–Rubincam effect, but is also a way to get an upper–bound order–of–magnitude estimate of the non–gravitational perturbations error budget. We can then combine linearly – following Eq. (9) – the rates obtained in such a way to estimate the impact of the uncertainty of each perturbation in the Lense–Thirring effect determination. The results obtained are reported in Tab. 5.

Table 5: Errors on the Lense–Thirring measurement due to the mismodelled non–gravitational perturbations.

Perturbation	$\frac{\delta\mu_{LT}}{\mu_{LT}}$
Yarkovsky-Rubincam	3×10^{-3}
Yarkovsky-Schach	6×10^{-4}
Asymmetric reflectivity	5×10^{-4}
Earth albedo	1×10^{-4}
Direct solar radiation pressure	5×10^{-4}

As we can see, the largest effect is that of the Earth Yarkovsky–Rubincam perturbation. This is due to the fact that all the perigee rate effects are depressed by the very small values of the coefficients c_3 and c_4, then the major contributions in the combination are due to the nodes of LAGEOS and LARES. It is also important to stress that, while the asymmetric reflectivity and solar Yarkovsky perturbations gives only long–term periodic effects in the analysed elements, those due to the Earth Rubincam perturbation are both secular and periodic in the nodal and perigee rates. Of course, when the resulting time–varying perturbations exhibit harmonic behavior with known relatively short periods, over an observational time span T_{obs} of some years, they could be fitted and removed from the signal – as for the tidal perturbations – with an improvement in the root–mean–square of the residuals. But this technique cannot be applied in the case of the secular non–gravitational perturbations in the satellites node and perigee.

To determine the error budget estimate of the Lense–Thirring effect measurement due to

the analysed non–gravitational perturbations we have conservatively added their contributions. We finally obtain

$$\frac{\delta\mu_{LT}}{\mu_{LT}}_{NG} \sim 5 \times 10^{-3}. \tag{15}$$

As we can see, the impact of the mismodelled non–gravitational perturbations uncertainties is below 1% of the relativistic parameter μ_{LT}, even with our conservative approach. Notice also that it is larger than the error by the static part of the geopotential.

We finally conclude this section underlining a few points

- It should be noticed that, according to certain pessimistic evaluations, the true, realistic errors in the perigees rates of LAGEOS–like satellites induced by various sources of systematic biases might even amount to 100% of their Lense–Thirring shifts. However, even in this case, the impact on our proposed configuration would amount to 2×10^{-3} thanks to the small coefficients with which the perigees of LAGEOS II and LARES enter the proposed combination.

- Moreover, the observational error in the LAGEOS II and LARES perigees, which are undoubtedly difficult to measure for low eccentric satellites as LAGEOS due to the small value of their eccentricity, would have an impact of the order of 1×10^{-4} by assuming an uncertainty of the order of 1 cm over 1 year in the satellite's position.

- Finally, preliminary estimates of the standard statistical error in the solve-for least square parameter μ_{LT}, based on simulations encompassing the present models of the time-dependent LAGEOS perturbations [*Iorio*, 2001; *Lucchesi,* 2001; 2002] and the noise level reported in the Lense-Thirring LAGEOS experiment, yield a value of the order of 10^{-3}.

So, it should not be unrealistic to predict a total uncertainty below 1%, according to the present–day force models.

3.2   A nodes-only combination

In order to avoid the use of the perigees, which, independently of the coefficients with which they would enter the observable, are more sensitive than the nodes to a large set of classical

gravitational and non–gravitational perturbations, the following alternative combination may be proposed as well

$$\delta\dot\Omega^{\text{LARES}} + k_1\delta\dot\Omega^{\text{LAGEOS}} + k_2\delta\dot\Omega^{\text{Ajisai}} + k_3\delta\dot\Omega^{\text{LAGEOS II}} = 62.7\mu_{\text{LT}}, \tag{16}$$

with

$$k_1 = 1.01, \tag{17}$$

$$k_2 = 4\times 10^{-5}, \tag{18}$$

$$k_3 = 3\times 10^{-3}. \tag{19}$$

Also in this case, the coefficients k_1, k_2 and k_3 have been calculated for the nominal values of the orbital parameters of LARES. Eq. (16) uses only the nodes of the three LAGEOS satellites and of Ajisai [*Iorio*, 2002] and cancels out the mismodelled contributions of J_2, J_4 and J_6. The relative error due to the remaining zonal harmonics of the geopotential would amount to 3×10^{-4}, as in the case of the previously proposed combination including the perigees of LAGEOS II and LARES. Moreover, the time–dependent part of the error budget would be mainly dominated by the nodes of LAGEOS and LARES. We notice that the Ajisai satellite is much more sensitive than the LAGEOS satellites to the non–gravitational perturbations. Indeed, its area–to–mass ratio is larger than that of the LAGEOS satellites. Nevertheless, the coefficient k_2 weighting the Ajisai contribution in the combination eq. (16) is just 4×10^{-5}, i.e. much smaller than the other weighting coefficients in eq. (16).

As shown in Fig. 5, the main drawback of eq. (16) would be its sensitivity to the orbital injection errors in the LARES inclination, contrary to eq. (9) and Fig. 3. Instead, regarding the semimajor axis of LARES, eq. (16) would be rather insensitive to its injection error.

4   The POLARES

In order to cope with practical launching costs it is currently under consideration the possibility of inserting the new LAGEOS-like satellite in a low altitude polar orbit with $i = 90$ deg and $a = 8,378$ km obtaining the so called POLARES [*Lucchesi and Paolozzi*, 2001]. The Lense–Thirring shift of its node would amount to 96.9 mas/y.
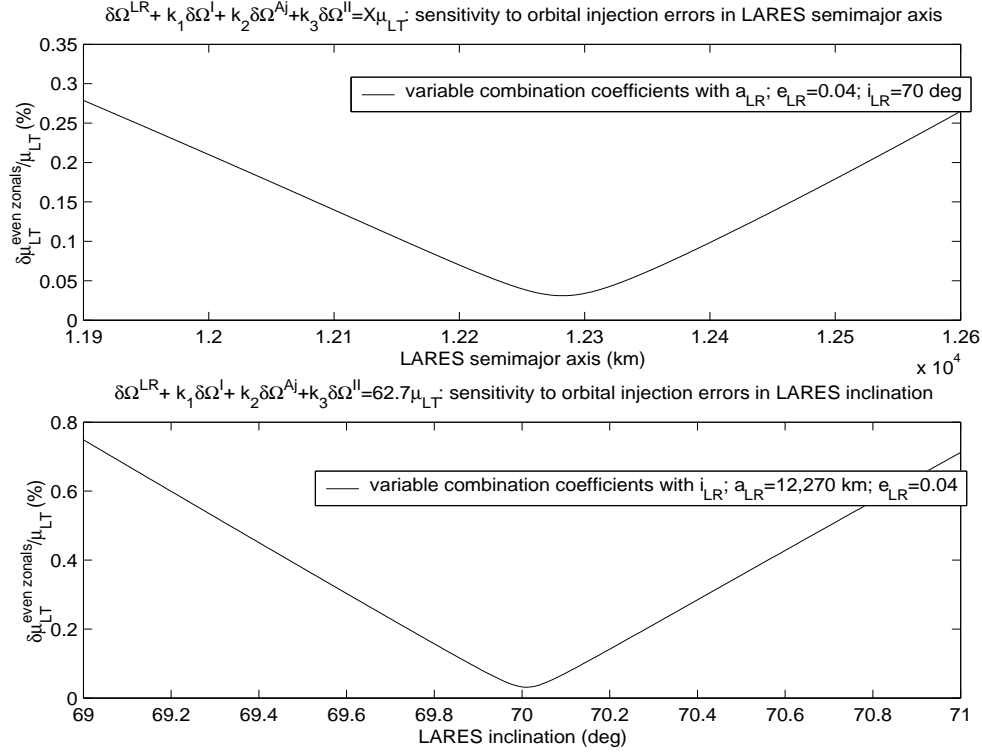
Figure 5: Influence of the injection errors in the LARES inclination on the error in the nodes-only combination due to the even zonal harmonics of the geopotential.

If we could obtain and keep exactly an inclination of 90 deg we would be able to use the POLARES node only because the classical mismodelled nodal precessions, which depend on $\cos i$, would vanish. However, the unavoidable injection errors in the POLARES inclination, in this case, would be greatly enhanced by the too low altitude in the sense that the systematic error due to the even zonal harmonics would blow up even for relatively small departures from the nominal values.

This is clearly shown by Fig. 6. Also in this case, this result has been obtained by adding in a root–sum–square fashion the correlated mismodelled classical nodal precessions with the EGM96 model up to degree $l = 20$. Moreover, it should be considered that, in this case, also the even zonal harmonics of degree higher than 20 would affect the systematic geopotential error.

As expected, for $i_{\rm PL} = 90$ deg the systematic zonal error vanishes. It turns out that even by including the POLARES in some combinations, this configuration would remain unfavorable.
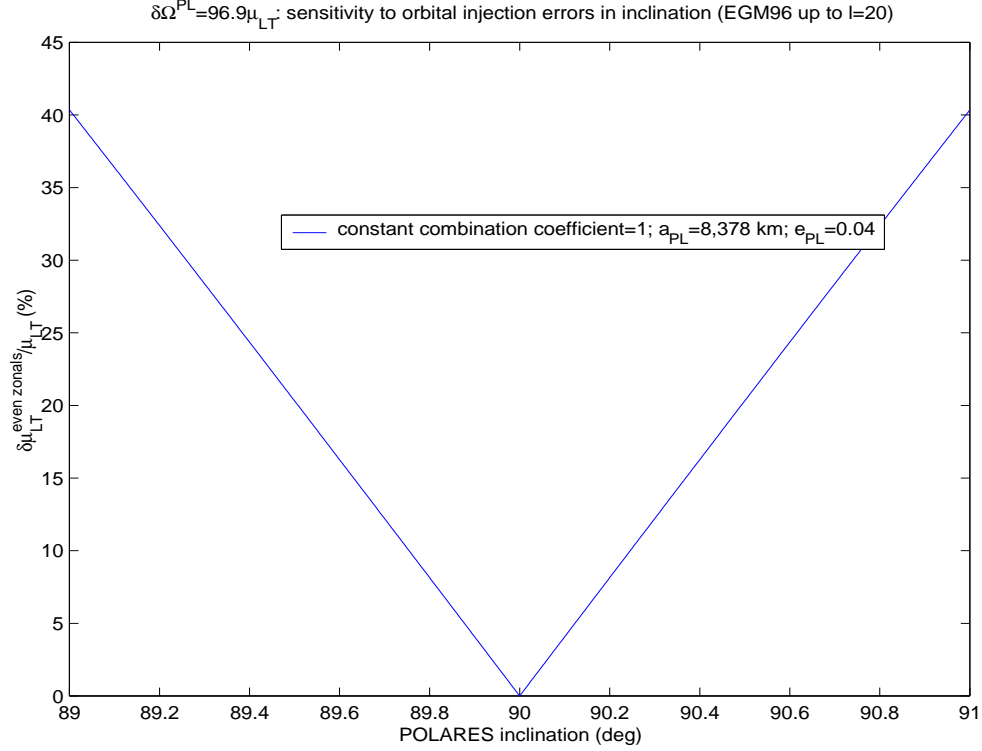
Figure 6: Influence of the injection errors in the POLARES inclination on the error in the node only induced by the even zonal coefficients of the geopotential.

However, the forthcoming new gravity models from CHAMP and GRACE will improve the error budget.

5 Conclusions

If analyzed from the point of view of the impact of the systematic error induced by the mis-modelling in the even zonal harmonics of the geopotential, which is the most important source of systematic error, the originally proposed LARES observable, consisting of the sum of the nodes of LAGEOS and LARES, is somehow sensitive to the possible departures of the original LARES orbital parameters from their nominal values due to orbital injection errors. The related systematic error could even raise to few percent, especially as far as the semimajor axis is concerned. It should be also considered that LARES could be put into an orbit with a low-cost launcher which, inevitably, would induce relatively large injection errors. Indeed, the

most expensive part of the implementation of the LARES mission would just be the launch and in such experiment, which would get a measurement of the Lense–Thirring effect with accuracy comparable to the Stanford GP-B mission [*Everitt et al.,* 2001], this could be a serious drawback.

The adoption of the alternative combined residuals proposed here, including also the node of LAGEOS II and the perigees of LAGEOS II and LARES, would reduce by about one order of magnitude the systematic error due to the even zonal harmonics of the geopotential decreasing from 0.3% to 0.02%, according to the present–day EGM96 gravity model, and would greatly reduce the sensitivity of such result to errors in the LARES orbital parameters. This would yield to less stringent requirements on the quality and the costs of the launcher to be adopted.

Preliminary estimates of the error budget, based on the present–day force models such as the EGM96 Earth gravity model, show that it would be possible to obtain a total error $\leq 1\%$. It is very important to notice that when the new data on the terrestrial gravitational field from the CHAMP and GRACE missions will be available, the systematic error due to the even zonal spherical harmonic coefficients of the geopotential will greatly reduce. The impact of the errors related to the quality of laser data will further reduce in the near future as well. However, a careful analysis of the error induced by the spin–dependent, non–gravitational thermal forces will be required together with an accurate modelling of the temporal variations of the satellites reflectivity coefficients. In particular, with the despinning of the satellites rotational period, more refined thermal models are needed in such a way to consider the equatorial component of the perturbing acceleration jointly with the component along the spin–axis direction, the only one necessary in the rapid–spin approximation [*Vokrouhlický and Farinella*, 1997]. This perturbative acceleration arises when the longitudinal gradient of temperature, induced by the slowed down rotation of the LAGEOS satellites, become significative. It is worth noticing that, with the proposed observable, the impact of the non–gravitational orbital perturbations is more relevant than that of the gravitational perturbations; this fact will be further enforced when the gravity models from CHAMP and GRACE will be available.

It should also be possible to adopt a combination involving only the nodes of LARES, LAGEOS, Ajisai and LAGEOS II. The systematic error due to the even zonal harmonics of the geopotential would be equal to the previous case in which the node of Ajisai would be

substituted by the perigees of LARES and LAGEOS II, but this combination turns out to be more sensitive to the orbital injection errors in the inclination of LARES.

The possibility of injecting LARES in a low polar orbit at 2,000 km of altitude in order to consider only its nodal rate would be, at the present level of knowledge of the Earth's gravity field, a not entirely satisfactory solution because, according to our evaluations based on EGM96, even small deviations from the projected inclination would lead to an error due to the even zonal harmonics of the geopotential of several percents. Anyway, this solution may be considered as a lower–cost approach to the Lense–Thirring effect determination in such a way to reduce the error budget below the $\sim 20\%$ uncertainty of the actual measurement obtained with LAGEOS and LAGEOS II [*Lucchesi*, 2002]. However, the systematic gravitational errors using a polar satellites will be reduced with the new gravity models from the CHAMP and GRACE missions.

The approach outlined here could also be useful for other precise general relativistic tests, as sketched in [*Iorio*, 2002] for the measurement of the gravitoelectric perigee advance. Moreover, in view of an "opportunistic" approach to general relativistic measurements, this strategy could easily be exploited in encompassing other satellites, proposed or planned to be launched in the near future for other scopes, if they will be useful for testing General Relativity as well.

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